

Study Abroad Proposal

As an undergraduate looking forward to a career in academia, I believe studying abroad is a great opportunity for personal growth.

When I first became a student in mathematics, I preferred learning a concept from only one standard textbook. Later I realized, that was my subconscious reaction to the overwhelming rigor at the heart of this subject: I was in constant fear of being confused by different treatments of the same material.

Yet, as I was building up knowledge to approach deeper concepts, I noticed that different interpretations of the same object are both useful and necessary towards a thorough understanding. This realization has later become my major motivation to apply for the study abroad program. As I am enjoying the thoughtfully structured courses in Caltech, I am also looking forward to being exposed to different approaches to the same topic. Courses conducted at universities abroad are likely to offer that aspect due to differences in teaching philosophy and choice of textbooks. Thus, I am excited about employing this opportunity to gain a deeper understanding at topics of my interest.

Along the same line of thoughts, studying abroad allows me to meet more friends with similar academic interest. From my SURF experience last summer, communicating ideas with people working on related problems is extremely inspiring. I would not have explored so many directions in solving my research question without the delightful discussions with my friends. During my time abroad, I plan to meet friends with shared academic interest through both lectures and regular department seminars. As I intend to work on problems concerning elliptic curves this coming summer, I am interested in the talks offered by Algebraic Geometry and Number Theory seminars in Cambridge, which fit my proposed schedule. Although such seminars usually target full-time research students, my prior experience of attending topic courses (Ma 191) will help me to make the most out of these highly specialized talks. When exposed to such comprehensive resources, I will be ready to get inspired and contribute to discussions.

Besides, I cherish studying abroad as a wonderful opportunity to immerse myself in a different culture. Last six years of living independently in foreign countries has fostered my passion for exploring “social norms” of a country. Personally, I believe they are the most intriguing subtleties of a culture, analogous to expressions which are difficult to be translated into another language. I plan to understand them by interacting with local students, for example, through clubs and societies. Since I am playing Baroque music in Caltech Chamber Music group, joining Cambridge University Baroque Ensemble or Chamber Orchestra at Edinburgh will be a good way to meet new people. Besides, enrolling in Fall is helpful since I can make friends with incoming students without accidentally “intruding” existing friend-groups. With these approaches, I can obtain knowledge of “social norms” that will not only feed my passion but also be beneficial for future collaboration with colleagues from different backgrounds.

Through studying abroad program, I see myself becoming someone more modest in front of the multifaceted aspects of math, and at the same time, someone who, in trying to understand a different culture, is more confident in communicating science to a diverse audience.

Program Fit

Cambridge, Michaelmas:

Michaelmas term is suitable for me because the classes offered match my academic interest. I have taken an introductory course in Algebraic Topology (Ma 109 a) during fall term sophomore year and I am currently (winter, sophomore) taking a reading course about applications of Algebraic Topology in theoretical physics. Thus, I am interested in continuing to learn Algebraic Topology in Cambridge and hopefully take Ma 151bc¹ after I come back. Besides, due to my strong interest in physics since high school, I am happy to see Cosmology and Quantum Mechanics offered in the Math Tripos. I plan to continue with Ph 125bc² in the following terms. The choice of Automata and Formal language is purely out of personal interest. As I have not yet taken courses related to discrete mathematics, it will be interesting to take it at the very university where Turing machine are introduced. Besides, I am also eager to explore the Supervision system, which is very different from the system at Caltech. Personally, I see it as a perfect opportunity to practice academic "autonomy" without the presence of strict deadlines.

Cambridge, Lent:

Lent term is suitable for me because the classes offered match my academic interest and could potentially be used to fulfill option requirements. Since my SURF project this summer (sophomore) is related to elliptic curves, I am interested to learn more in Cambridge. Depending on the materials covered at Caltech in Ma 130³ next year, it is possible to continue with Ma 130c with some extra reading. Due to my interest in quantum mechanics, I am glad to find Quantum Information and Computation in the math tripos. Since I do not have much prior knowledge in theoretical computer science but I do have some introduction to quantum mechanics, this course is very suitable for me. I choose to take coding and cryptography due to my interest in Number Theory. In fact, I have learned some basic cryptography methods in Ma 7, an introductory Number Theory course. I want to take Logic and Set Theory to fulfill my option requirement in Logic at Caltech. The only concern for Lent term is its compatibility with my schedule. Since I plan to take three tracks in Junior year, including Ma 130, Ma 151 and Ph125, it can be more difficult to catch up after winter term.

¹ Algebraic & Differential Topology

² Quantum Mechanics

³ Geometry

Proposed Course List:

Cambridge, Michaelmas

Total CIT Units for term abroad: 36

Course by Correspondence: N.A.

1. Automata and Formal Language

Tripos: Mathematics

Part: II, C

Term: Michaelmas

Number of lectures: 24

Lecture times: Tue, Thurs, Sat 12pm-1pm

Caltech units: 9

Caltech evaluator: Chris Umans

Type of Caltech Credit: General

CIT equivalent course: CS 21: Decidability and Tractability

Course description: The course deals with three basic ideas: the idea of computability for a function; what it means for a function to be non-computable; and how computational power depends on programming constructs. Such questions appear straightforward, but they are not. In seeking answers students will meet fundamental and recurring concepts such as state, transition, non-determinism, reduction and undecidability. They remain important in current theoretical computer science even as technology changes from day to day. Three classes of models are used to illustrate some fundamental aspect of computation, most of which were developed long before computers existed.

2. Cosmology

Tripos: Mathematics

Part: II, C

Term: Michaelmas

Number of lectures: 24

Lecture times: Mon, Wed, Fri 9am-10am

Caltech units: 9

Caltech evaluator: Saul Teukolsky

Type of Caltech Credit: General

CIT equivalent course: Ph 136c: Applications of Classical Physics

Course description: The principal aim of this course is to provide the outlines of our current understanding of the evolution of the universe, from the Big Bang to the present day. An understanding of the early universe requires some knowledge of statistical mechanics, which is therefore taught as part of the course. Although modern cosmology is based on Einstein's theory of gravity, General Relativity, the basic equations actually follow from Newtonian gravity, given the equivalence of mass and energy via $E = mc^2$. The course will begin with a derivation of these equations and an investigation of their cosmological consequences. Statistical Mechanics will then be introduced and applied to the early universe. The course prerequisites

are a knowledge of Newtonian dynamics and the rudiments of Quantum Mechanics and Special Relativity.

3. Algebraic Topology

Tripos: Mathematics

Part: II, D

Term: Michaelmas

Number of lectures: 24

Lecture times: Mon, Wed, Fri 12pm-1pm

Caltech units: 9

Caltech evaluator: Markovic Vladimir

Type of Caltech Credit: Option

CIT equivalent course: Ma 151a: Algebraic and Differential Topology

Course description: Two particular algebraic invariants are studied in this course: the fundamental group, and the simplicial homology groups. Of these, the former is easier to define, but hard to calculate except in a few particular cases; the latter requires the erection of a considerable amount of machinery before it can even be defined, but once this is done it becomes relatively easy to calculate. The course concludes with a classic example of the application of simplicial homology: the classification of all compact 2-manifolds up to homeomorphism.

4. Principles of Quantum Mechanics

Tripos: Mathematics

Part: II, D

Term: Michaelmas

Number of lectures: 24

Lecture times: Mon, Wed, Fri 11am-12pm

Caltech units: 9

Caltech evaluator: Mark B. Wise

Type of Caltech Credit: General

CIT equivalent course: Ph 125a: Quantum Mechanics

Course description: This course develops the principles and ideas of quantum mechanics in a way which emphasizes the essential mathematical structure, while also laying the foundations for a proper understanding of atomic and sub-atomic phenomena. In contrast to the introductory treatment given in Part IB, which is based entirely on wavefunctions and the Schrodinger equation, observables are presented as linear operators acting on vector spaces of states. This new approach has practical as well as aesthetic advantages, leading to elegant and concise algebraic solutions of problems such as the harmonic oscillator and the quantum theory of angular momentum. Some of the other key aspects of quantum behavior that are treated include: intrinsic spin, multi-particle systems, symmetries, and their implications. Perturbation theory techniques, which are indispensable for realistic applications, are also discussed. The course ends by examining in more detail the inherently probabilistic nature of quantum mechanics, as illustrated by Bell's inequality and related ideas.

Cambridge, Lent

Total CIT Units for term abroad: 36

Course by Correspondence: N.A.

1. Quantum Information and Computation

Tripos: Mathematics

Part: II, C

Term: Lent

Number of lectures: 24

Lecture times: Mon, Wed, Fri 11am-12pm

Caltech units: 9

Caltech evaluator: Fernando Brandao

Type of Caltech Credit: General

CIT equivalent course: Ph 219a: Quantum Computation

Course description: We will begin by expounding the postulates of quantum mechanics in this setting (using Dirac notation) and then immediately make connections to information (quantum states viewed as information carriers, quantum teleportation) and computation (notion of qubits and quantum gates). Then we will discuss quantum cryptography (quantum key distribution), and quantum computing, culminating in an exposition of principal quantum algorithms, including the Deutsch–Jozsa algorithm, Grover’s searching algorithm and an overview of Shor’s quantum factoring algorithm. The course is cross-disciplinary in its conceptual ingredients and will be of interest to pure and applied mathematicians alike.

2. Coding and Cryptography

Tripos: Mathematics

Part: II, C

Term: Lent

Number of lectures: 24

Lecture times: Mon, Wed, Fri 10am-11am

Caltech units: 9

Caltech evaluator: Thomas Vidick

Type of Caltech Credit: General

CIT equivalent course: CS152: Introduction to Cryptography

Course description: When we transmit any sort of message errors will occur. Coding theory provides mathematical techniques for ensuring that the message can still be read correctly. Since World War II it has been realized that the theory is closely linked to cryptography – that is to techniques intended to keep messages secret. This course will be a gently paced introduction to these two commercially important subjects concentrating mainly on coding theory.

Discrete probability theory enters the course as a way of modeling both message sources and (noisy) communication channels. It is also used to prove the existence of good codes. In contrast, the construction of explicit codes and cryptosystems relies on techniques from

algebra. Some of the algebra should already be familiar – Euclid’s Algorithm, modular arithmetic, polynomials and so on – but there are no essential prerequisites.

3. Algebraic Geometry

Tripos: Mathematics

Part: II, D

Term: Lent

Number of lectures: 24

Lecture times: Mon, Wed, Fri 12pm-1pm

Caltech units: 9

Caltech evaluator: Christopher J. Campbell

Type of Caltech Credit: Option

CIT equivalent course: Ma 130: Algebraic Geometry

Course description: This course is an introduction to the basic ideas of algebraic geometry (affine and projective spaces, varieties), followed by a more detailed study of algebraic curves. We will develop the basic tools for understanding the properties of algebraic curves, and apply these at the end of the course to the beautiful theory of elliptic curves, which among other things played an essential part in the proof of Fermat’s Last Theorem!

4. Logic and Set Theory

Tripos: Mathematics

Part: II, D

Term: Lent

Number of lectures: 24

Lecture times: Mon, Wed, Fri 9am-10am

Caltech units: 9

Caltech evaluator: Alexander S. Kechris

Type of Caltech Credit: Option

CIT equivalent course: Ma 116a: Mathematical Logic and Axiomatic Set Theory

Course description: The course falls into three main parts. One part develops the notions of validity and provability in formal logic, culminating in the Completeness Theorem, which asserts that these two notions coincide. Another part is concerned with ordinals and cardinals: these are notions that generalize the ideas of size and counting to the infinite. The final part is an introduction to formal set theory, where one makes precise the idea of a ‘universe of sets’, and studies its structure.